Klein Backscattering and Fabry-Pérot Interference in Graphene Heterojunctions

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We present a theory of quantum-coherent transport through a lateral p-n-p structure in graphene, which fully accounts for the interference of forward and backward scattering on the p-n interfaces. The backreflection amplitude changes sign at zero incidence angle because of the Klein phenomenon, adding a phase $\pi$ to the interference fringes. The contributions of the two p-n interfaces to the phase of the interference cancel with each other at zero magnetic field, but become imbalanced at a finite field. The resulting half-period shift in the Fabry-Pérot fringe pattern, induced by a relatively weak magnetic field, can provide a clear signature of Klein scattering in graphene. This effect is shown to be robust in the presence of spatially inhomogeneous potential of moderate strength.

As illustrated in Fig. 1, the contribution $\Delta \theta_1 + \Delta \theta_2$ to the net phase can be altered by a magnetic field. At zero $B$ the incidence angles at interfaces 1 and 2 have opposite signs, and thus the jumps in $\Delta \theta_{1(2)}$ cancel. However, for curved electron trajectories at a finite $B$, the signs of the incidence angles can be made equal. Indeed, because of translational invariance along the p-n interface, in the presence of a magnetic field the y component of electron momentum varies in space as $p_y(x) = p_y - eBx$, where $p_y$ is the conserved canonical momentum component that labels different trajectories. For the incidence angles at interfaces 1 and 2 to be of equal sign, $p_y(x_1)$ and $p_y(x_2)$ must have opposite signs, which happens when

$$-eBL/2 < p_y < eBL/2. \quad (2)$$

In this case the net backreflection phase $\Delta \theta_1 + \Delta \theta_2$ in (1) equals $\pi$. As we shall see, the backreflection phase manifests itself as half a period shift of the FP fringe contrast. This phase shift develops for the field strength such that the...
range (2) exceeds the Klein collimation range in which the p-n interface is transparent.

One useful feature of the jump in backreflection phase is that it is less momentum-selective than collimated transmission. A potential difficulty, however, is that the interference of scattering on two p-n interfaces can be sensitive to disorder. Below we will investigate the dependence of the FP contrast on magnetic field in the presence of large-scale spatial fluctuations. We find that, while the FP fringe contrast is suppressed, the 1/2-period shift, controlled by backreflection, remains surprisingly robust. Even at a relatively high disorder strength, when the FP contrast is zero. Transmission, evaluated from the numerical solution of Eq. (6), exhibits resonances as a function of the FP interference, occurring when the separation between the crossings. This condition yields \( x = \pm x_s \), Eq. (3). A Landau-Zener transition at the first crossing creates a coherent superposition of the diabatic states that can interfere at the second crossing. This so-called Stueckelberg interference (see Ref. [17] and references therein), which can be constructive or destructive, is described by an oscillatory function of the phase \( \Delta \theta = -2 \int_{x_s}^{x_s} U(x) dx = \frac{4}{\hbar} e x \), gained between the crossings. The locations of interference fringes, determined from the conditions \( \Delta \theta = 2\pi n \) with \( n = 1, 2, \ldots \), are \( x_n = (3\pi n/2)^{1/3} e_s \), which agrees with the positions of the fringes seen in Fig. 2.

For \( B = 0 \), the suppression of oscillations near zero \( p_y \) can be linked to the absence of Landau-Zener transitions at vanishing level splitting. In the scattering picture, this is nothing else than the Klein phenomenon of perfect transmission at normal incidence. At finite \( B \), the oscillations

\[
\psi(x, y) = e^{i p_y x} \psi(x), \\
\text{and solve the one-dimensional Schrödinger equation}
\]

\[
i \partial_x \psi = \left( U(x) \sigma_3 - i (p_y - eBx) \sigma_1 \right) \psi,
\]

where without loss of generality we set the Fermi energy equal to zero. Transmission, evaluated from the numerical solution of Eq. (6), exhibits resonances as a function of momentum and potential depth, shown in Fig. 2. We note a drastic difference between the results at zero \( B \), similar to those of Refs. [15,16], and the results at finite \( B \).

To understand the behavior of transmission, it is instructive to view the differential equation (6) as a fictitious time-dependent Schrödinger evolution of a two-level system, E(\( x \)) playing the role of time. In this analogy, the system is driven through an avoided level crossing with splitting \(-2i(p_y - eBx)\) at the crossing times determined by degeneracy of the diabatic states, \( U(x) = ax^2 - e = 0 \). This condition yields \( x = \pm x_s \), Eq. (3). A Landau-Zener transition at the first crossing creates a coherent superposition of the diabatic states that can interfere at the second crossing. This so-called Stueckelberg interference (see Ref. [17] and references therein), which can be constructive or destructive, is described by an oscillatory function of the phase \( \Delta \theta = -2 \int_{x_s}^{x_s} U(x) dx = \frac{4}{\hbar} e x \), gained between the crossings. The locations of interference fringes, determined from the conditions \( \Delta \theta = 2\pi n \) with \( n = 1, 2, \ldots \), are \( x_n = (3\pi n/2)^{1/3} e_s \), which agrees with the positions of the fringes seen in Fig. 2.

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where one of the level splittings $p_y \pm eBx_e$ vanishes, suppressing one of the Landau-Zener transitions. In terms of electron motion, this condition is equivalent to the requirement of normal incidence on either of the interfaces $(3)$, giving
\[ p_y = \pm eB\sqrt{\epsilon/a}, \] (7)
which is the black parabola drawn in Fig. 2(b). Indeed, fringes disappear on this line; upon crossing the line, the maxima and minima of fringes interchange, indicating a $\pi$ phase shift in the phase of fringe contrast.

A more refined description can be obtained from a quasiclassical solution [15] with position-dependent momentum $p_x(x) = \sqrt{U^2(x) - \tilde{p}_y^2(x)}$. The turning points, defined by $p_x = 0$, are arranged as $x_1 < x_2 < x_3$, with $x_2$ and $x_3$ equal to
\[ \sqrt{\epsilon + b^2} - p_y + b, \quad \sqrt{\epsilon + b^2} + p_y - b, \] (8)
where $b = \frac{1}{2} eB$ and $x_{2(1)}(p_y) = -x_{2(2)}(-p_y)$. Hereafter we set $a = 1$, restoring physical units later. Remarkably, the conditions $x_1 < x < x_2$ and $x_2 < x < x_3$, which correspond to one of the $p$-$n$ interfaces becoming transparent because of the Klein phenomenon, yield a relation between $p_x$ and $\epsilon$ which is identical to Eq. (7) found above.

The classically forbidden regions $x_1 < x < x_2$ and $x_2 < x < x_3$, where $p_x$ is imaginary, correspond to the Klein barriers at the interfaces 1 and 2. Denoting the corresponding transmission coefficients as $t_1$ and $t_2$, we can write the net transmission of the entire $p$-$n$-$p$ structure in a general Fabry-Perot form
\[ T(p_y) = \frac{t_1 t_2}{|1 - t_1 t_2 e^{i\Delta \theta}|^2}, \] (9)
where $r_{1(2)} = 1 - t_{1(2)}$ are the reflection coefficients, and the phase $\Delta \theta$ is a sum of the WKB part and the phases of the reflection amplitudes, Eq. (1).

The transmission amplitudes $t_{1(2)}$ can be evaluated in the WKB tunneling approximation:
\[ t_1 = e^{-2i\int_{x_1}^{x_1'} p_x(x')dx'} = e^{-\lambda(p_y - \epsilon B x_e)}^2, \quad \lambda = \frac{\pi}{2ax_e}, \] (10)
with the integral computed by linearizing $U(x)$ near $x = x_e$. Similarly, linearizing $U(x)$ near $x = -x_e$, we find $t_2 = e^{-\lambda(p_y + \epsilon B x_e)}^2$. Thus, the reflection amplitudes are
\[ \text{sgn}(p_y \pm eBx_e) e^{i\theta_{in}(p_y)} \sqrt{1 - e^{-\lambda(p_y \pm \epsilon Bx_e)^2}}, \] (11)
where we factored the sign, responsible for the phase jump, and a regular part of the phase $e^{i\theta_{in}}$, as follows from analyticity in $p_y$. Because the WKB treatment is exact for linear potentials [18], and the transmission $(10)$ is exponentially small unless $|p_y \pm eBx_e| \leq \lambda^{-1/2}$, the linearization of $U(x)$ used to evaluate the integral in $(10)$ gives accurate results for the energies of interest, $\epsilon \approx \epsilon_e$.

The dispersion of the resonances in Fig. 2 can be understood from the momentum dependence of the quasiclassical WKB phase (for simplicity, we set $B = 0$):
\[ \theta_{WKB} = \int_{x_1}^{x_2} p_x(x')dx' = \frac{4}{3} \sqrt{\epsilon} - \frac{p_y^2}{2\epsilon^{1/2}} \log \frac{\epsilon}{|p_y|}, \] (12)
where an expansion in the parameter $|p_y/\epsilon| \ll 1$ is legitimate because Klein collimation restricts transmission to $\Delta p_y \sim \lambda^{-1/2}$. The quantization condition $\theta_{12} = \pi(n + \frac{1}{2})$ gives the resonance energies $\epsilon_n(p_y)$ dispersing as in Fig. 2.

To summarize, the FP model $(9)$ is in full agreement with our numerical results. In particular, it explains the striking difference between the behavior at zero and finite $B$, as well as the phase shift of the fringe pattern, resulting from a sign change of the reflection amplitudes, Eq. $(11)$.

These results can now be applied to analyze conductance and resistance, given by
\[ R = G^{-1}, \quad G = \frac{4e^2}{h} W \int_{-\infty}^{\infty} T(p_y) \frac{dp_y}{2\pi}, \] (13)
where $W$ is the width of the $p$-$n$-$p$ structure (see Fig. 1). As illustrated in Fig. 3, the resistance exhibits fringes which obey the $n^{2/3}$ scaling, as expected from the phase dependence on $\epsilon$, Eq. $(12)$. Somewhat surprisingly, the integral over $p_y$ in $(13)$ yields a fairly high fringe contrast in $R$. This result is consistent with the fact that Klein collimation effectively restricts the integral to the range of $p_y$ where the resonances, Eq. $(12)$, are nondispersive.

In the presence of the magnetic field, alongside the overall increase in resistance, we observe that the fringes shift up in $\epsilon$ by approximately half a period (see Fig. 4). This shift, which is a direct consequence of the $\pi$-shift of the reflection phase discussed above, fully develops in the fields $B \sim 0.4B_e$. For the parameter values used above, Eq. $(5)$, we find a value of about $0.1 \text{T}$, which is well below the fields characteristic for magnetoresistance [11,12].

![FIG. 3 (color online). Fringes in resistance (13) at $B = 0$, plotted in the units of $R_e = (x_e/W)h/4e^2$ (blue line). Inset illustrates a $n^{2/3}$ scaling for the maxima and minima of $R$, which is consistent with the $\epsilon^{3/2}$ dependence of the WKB phase (12). Averaging over smooth potential fluctuations, described by a sum of a few harmonics, suppresses fringe contrast (red dashed line). Here we use Eq. (14) with $\sum |a_m| = 3\epsilon_e$.](image-url)
The effect of large-scale potential fluctuations, either intrinsic [19] or induced by variable distance to gates, can be analyzed by averaging the conductance in (13) over random offsets in potential depth $e$:

$$\langle G \rangle = \int G(\varepsilon - \sum_m a_m \cos(m\phi + \phi_m)) d\phi / 2\pi.$$  

(14)

This simple model describes a smooth inhomogeneity with correlation length larger than the $p$-$n$ interface separation $L$, but much shorter than the structure width $W$. The averaging procedure (14), applied to our numerical results, makes the fringes aperiodic and suppresses the contrast (see red dashed line in Fig. 3). However, the $\pi$ phase shift induced by magnetic field remains clearly discernible even for relatively strong fluctuations [20].

At even stronger randomness, the FP transmission (9) can be replaced by its phase-averaged value

$$\langle T \rangle = \frac{t_1 t_2}{1 - r_1 r_2} = \frac{1}{\sqrt{e^{(p_r e B x_x)^2} + e^{(p_f e B x_x)^2}} - 1}. $$  

(15)

Plugged in (13), it yields magnetoresistance with characteristic $B \sim B_0$, identical to that discussed in [4]. The resulting exponential suppression of conductance of course would hold only in the absence of short-range disorder.

Conspicuously, the resistance data [11,12] feature aperiodic oscillations in gate voltage, observed above the point where the sign of carriers beneath the top gate is reversed. This is the same region where strong FP fringes are expected for an ideal system. The energy scale of the oscillations reported in Ref. [11], converted from gate voltage using $\delta \varepsilon / \delta V_g = \frac{1}{\sqrt{W}}$, is about $\delta \varepsilon \sim 30 \text{ mV}$, which is only a few times larger than the period of $0.8 e_0 = 11 \text{ mV}$ found above (Figs. 3 and 4). Could these oscillations, or those seen in [12], be the FP fringes contaminated by disorder? Comparison to the behavior of the FP contrast in the presence of magnetic field, in particular, to the $\pi$ phase shift (Fig. 4), may help to clarify this.

In summary, Fabry-Pérot interference in the Klein scattering regime is found to be sensitive to the phase of the reflection amplitude that exhibits a jump by $\pi$ near zero incidence angle. This leads to half a period shift of interference fringes in the presence of a relatively weak magnetic field, a new effect that can help to identify the Klein phenomenon in graphene.

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Note added.—Recently, we became aware of the work [21] which reports unambiguous FP oscillations in a $p$-$n$-$p$ structure. At low fields $B \leq 1 \text{ T}$ the behavior of the observed fringes is consistent with our predictions, however at higher fields the fringes are found to continuously transform into Shubnikov-deHaas oscillations instead of being suppressed. This indicates coexistence of momentum conserving and impurity assisted contributions to transport, which are dominant at low and high $B$, respectively. The FP and ShD oscillations can be understood on equal footing from quantization of periodic orbits of an electron bouncing between $p$-$n$ boundaries (to be published).